Damage Assessment of Structures with Uncertainty

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ABSTRACT

Evolutionary algorithms are suitable to solve damage identification problems in a multiobjective context. However, the performance of these methods can deteriorate quickly with increasing noise intensities originating numerous uncertainties. In this work, a statistic structural damage detection method formulated in a multiobjective context is proposed, taking into account the uncertainties existing. The presented method is verified by a number of simulated damage scenarios. The effects of noise on damage detection are investigated.

INTRODUCTION

Model updating methods based on structural vibration data have been developed and applied to identify structural damage in civil engineering. Structural damage is located and quantified by minimizing the response discrepancies between a finite element (FE) model and its corresponding structure before and after damage [1, 2]. To do this, an objective or error function dependent on the agreement between numerical predictions and experimental data is defined and optimized.

However, in most real-world problems the relative importance among objectives is not generally known until the system's best capabilities are determined and trade-offs between the objectives are fully understood. Furthermore, objectives under consideration come into conflict with each other, and optimizing a particular solution with respect to a single objective can lead to unacceptable results regarding the other objectives [3, 4]. With the purpose of improving the robustness and performance of the procedure, all the objectives must be treated as a whole, instead of optimizing them independently. Evolutionary algorithms are a class of stochastic search methods that have been found to be very efficient and effective in solving complex multiobjective problems where conventional optimization tools fail to work well.

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On the other hand, most of the damage identification problems are presented under the effects of different degrees of uncertainties, known as noise. This noise has to be understood as the multiple and uncontrollable events that are able to twist the result of an analysis, degrading the algorithm's performance with a premature convergence to sub-optimal solutions. Therefore, the performance of MOEA deteriorates quickly with increasing noise intensities. Besides this, the design variables may change after optimization, and the quality of the optimal solution should be robust against environmental changes or deviations from the optimal point [5].

A strategy is proposed in this work to improve the performance and robustness of the damage detection methods in noisy problems. In this strategy, solutions should still work satisfactorily when the design variables change slightly.

OBJECTIVE FUNCTIONS

The objective functions selected for the FE model updating method have to reflect the deviation between the numerical prediction and the real behavior of the structure. Therefore, they should be formulated in terms of the discrepancy between FE and experimental quantities. In this work, due to their high performance, the following functions have been chosen [6]:

$$F_1 = 1 - \prod_{j=1}^{m} MTMAC_j = 1 - \prod_{j=1}^{m} \frac{MAC(\{\phi_{num\ j}\},\{\phi_{exp\ j}\})}{1 + \left|\frac{\lambda_{exp\ j} - \lambda_{num\ j}}{\lambda_{exp\ j} + \lambda_{num\ j}}\right|} \quad (1)$$

$$F_2 = 1 - \prod_{j=1}^{m} MACFLEX_j = 1 - \prod_{j=1}^{m} \frac{\left|\{F_{num\ j}\}^T\{F_{exp\ j}\}\right|^2}{\left(\{F_{num\ j}\}^T\{F_{num\ j}\}\right)\left(\{F_{exp\ j}\}^T\{F_{exp\ j}\}\right)} \quad (2)$$

where $\{\psi_j\}$ is the j-th mode shape, $\lambda j=(2\pi f_j)^2$ where f_j is the eigenfrequency corresponding to j-th mode and $\{Fj\}$ is the flexibility vector, collecting the diagonal terms of the flexibility matrix corresponding to the j-th mode; MAC is the modal assurance criterion [7] and the subscript num and exp are respectively referred to numerical and experimental values. Both functions take values between zero and one.

Noise

Ideally, evolutionary algorithms should work on the expected objective functions and not be misled due to the presence of noise. In some works [4], noise has been incorporated as an additive perturbation to the original objective functions, with the purpose of including in this perturbation all possible kind of noise.

On the other hand, in other works [1] noise has also been incorporated affecting directly to the so called *elemental stiffness parameter*, so that noise appears not only in the measured data but also in the FE model. In order to study this, noise has been introduced as follows:

$$K = K \cdot (1 + \chi_1) \quad (3)$$

where K is the stiffness of the model and χ_1 represent the random noise with zero mean and variance σ^2. Finally, in this study, quasi-experimental data have been used, since the experimental data are originated numerically, so the noise has also been introduced in the quasi-experimental frequencies (λ_{exp}) as follows:

$$\lambda_{exp} = \lambda_{exp} \cdot (1 + \chi_2) \qquad (4)$$

EVOLUTIONARY MULTIOBJECTIVE OPTIMIZATION

Since objectives under consideration come into conflict with each other, instead a unique solution there will be a set of optimal solutions, known as Pareto-optimal solutions. In this case, the notion of optimality is not so obvious since does not exist any solution that minimizes all individual objectives simultaneously.

Evolutionary algorithms, such as genetic algorithms (GA), have been demonstrated to be very powerful and generally applicable not only to solve complex single objective problems, but also multiobjective problems, since in a single run of the algorithm several points of the Pareto-optimal set are found.

GA basic procedure is to randomly initialize a population of solutions and then to improve the solutions through repetitive operations of crossover, mutation and inversion. This theory was developed in the engineering area by Goldberg's work [8].

Modified Non-dominated Sorting Genetic Algorithm II

In this work, the Modified Non-dominated Sorting Genetic Algorithm (MNSGA-II) algorithm has been used. This algorithm is an alternative to NSGA-II, proposed by Babbar et al [8], with the purpose of improving some of its deficiencies. The main idea is to introduce explicit averaging into NSGA-II and to modify the non-dominated sorting procedure to allow seemingly dominated solutions into the first non-dominated front.

Although explicit averaging can reduce the uncertainty in the selection process, it is not feasible to use large number of samples. Since the number of samples is limited, it is inevitable that the first non-dominated front will comprise of both dominated and non-dominated solutions. To prevent the loss of potentially useful solutions, Babbar et al [8] suggested a clustering mechanism to induce solutions from the inferior non-dominated fronts into the first layer. This mechanism works by comparing the distances between solutions from the first non-dominated front and the other fronts.

A higher ranked solution F_a is re-assigned to the first non-dominated front if the following criterion is satisfied for any arbitrary objective,

$$\left|f_{a,i} - f_{b,i}\right| < k\sqrt{\frac{v_{a,i} + v_{b,i}}{2}} \qquad (5)$$

where $f_{b,i}$ is the i-th objective of solution F_b from the first non-dominated front, $v_{a,i}$ is the variance of F_a updated during the averaging process and k is the neighborhood restriction factor. The rationale is that it is very likely for a perceived inferior solution located in close proximity to a perceived non-dominated solution in the objective space to be a true non-dominated solution.

By assigning the first rank to these otherwise higher ranked solutions, truly good solutions are given the reprieve necessary to survive the selection process. The number of solutions present in the first non-dominated front is highly dependent on the setting of k; augmenting selection pressure while reducing k. MNSGAII applies a simulated annealing inspired adaptive scheme that reduces k over generations. The equation that governs the behavior of k is given by

$$K = C \cdot \left(1 - \exp\left(-\frac{\beta}{t}\right)\right) \qquad (6)$$

where C determines the largest setting of k and β controls the rate at which k reduces with t. Initially, k is large and more dominated solutions will be accepted into first non-dominated front. The rationale is that the reliability of the solution will increase with time through explicit averaging, allowing the algorithm to sort the solutions into the different layers of non-dominated fronts with a greater degree of certainty. At the end of the evolutionary process, the clustering mechanism is applied once again, this time to remove solutions that are significantly different from the other archived solutions.

Robustness

Working only with expected objective functions, solutions with high objective variance might be considered as robust since deviations from the true objective functions might cancel one with other in the target point. In this sense, the robustness of the method might be improved considerably by including, additionally, objective functions depending on variance as separate optimization criteria. In this way, the problem to be solved would be a four-objective optimization problem, in which the other two objective functions are selected from the following:

$$F_3 = \frac{\sigma_{F_1}}{\bar{\sigma}} \qquad (7)$$

$$F_4 = \frac{\sigma_{F_2}}{\bar{\sigma}} \qquad (8)$$

where σ_{F1} and σ_{F2} are the standard deviations obtained from the objective functions values at the sampling points used to get the averaged objective functions, and $\bar{\sigma}$ is the average of the standard deviations calculated for each one of the design variables j at the sampling points.

PROBABILITY OF DAMAGE EXISTENCE (PDE)

The PDE can be estimated from the statistical distributions of the stiffness parameter of the undamaged and damaged state [1]. The basic idea is to compute the probability of an elemental stiffness parameter at a confidence level, defining the interval of the healthy stiffness parameter, $\Omega(\alpha_i,\mu)$, so that the probability of α_i contained within the interval is μ:

$$\mathrm{prob}\big(x_\alpha \epsilon \Omega(\alpha_i, \mu)\big) = \mathrm{prob}(L_\Omega \leq x_\alpha < \infty) = \mu \qquad (9)$$

where L_Ω is the lower bound of the interval $\Omega(\alpha_i,\mu)$, which depends on the required confidence level. In this study, μ is set to 95%, thus $L_\Omega = E(\alpha_i) - 1.645*\sigma(\alpha_i)$, which means that there is a probability of 95% that the healthy stiffness parameter falls in the range of $[E(\alpha_i) - 1.645*\sigma(\alpha_i), \infty)$.

In the same way, an interval can be defined for the stiffness parameter of the damaged state ($\acute{\alpha}_i$). Thus, the PDE is defined as that of $\acute{\alpha}_i$ not within the 95% confidence healthy interval $\Omega(\alpha_i, 0.95)$. Thus, the PDE of an element i is:

$$p_d^i = 1 - \mathrm{prob}\big(x_{\tilde{\alpha}} \epsilon \Omega(\alpha_i, 0.95)\big) = 1 - \mathrm{prob}(L_\Omega \leq x_{\tilde{\alpha}} < \infty) \qquad (10)$$

PDE is a value between 0 and 1. It is apparent that if the PDE is close to 1, the damage of the element is most likely; otherwise, if the PDE is close to 0, the damage of the element is very unlikely.

NUMERICAL SIMULATION STUDY

In order to investigate the performance of the methodology proposed here, several numerical simulations on a simply supported beam have been made. In all the studies performed, a crossover probability of 0.8 and a mutation probability of 0.01 have been assumed for the GA. In the same way, a binary encoding has been used for the chromosomes of each individual of the population. Each design variable or damage variable $d_i \in [0,1]$ has been coded into a 3-bit binary number obtaining a resolution of 0.125, which is acceptable for a suitable estimation of damage. Once the optimal chromosome has been found, the damage values are obtained through the inverse of the decoded values.

For all cases, the exact solution is compared with the solution by the proposed multi-objective approach. In order to decrease the influence of random effects, 5 independent runs were performed per test problem to produce the mean ± one sample standard deviation plot.

The problem for a comparative investigation consists in identifying damage for a simply supported concrete beam of length L=6 m and rectangular cross section b x h = 0.25 m x 0.2 m. For numerical analysis purposes the beam was divided into 10 two-dimensional beam elements, resulting in a chromosome of 30 bits length. The beam was assumed to have a Young´s modulus E of 30 GPa and a density ρ of 2500 kg/m^3.

The beam was subjected to a simple (Figure 1) and a multiple simulated damage scenario (Figure 2) of complex identification. The "measured" dynamic responses of the beam before and after damage were generated previously. The baseline finite element model of the beam was created using Euler-Bernoulli planar elements with two degrees of freedom per node.

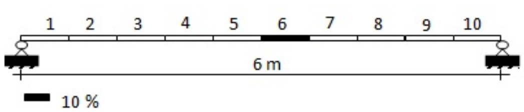

Figure 1. Simple damage scenario for the numerical beam.

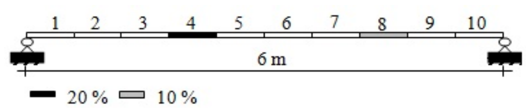

Figure 2. Multiple damage scenario for the numerical beam.

Since the present example is a numerical simulation, the baseline values are perfectly known and, therefore, the possible effect of poorly estimated baseline values is removed from this study. Any parameter estimation differing from the baseline value might be associated with damage.

To be more consistent with the field test conditions and to check the robustness of the proposed procedures, only the four lowest vibration modes were considered. Furthermore, two different levels of noise (5% and 20%) were included according with Eq. (3) and (4).

As commented above, two damage scenarios were tested, both four-objective problems (Eq. (1), (2), (7) and (8)) solved with MNSGA-II method also described above. Figures 3 and 4 show the damage distribution for the beam problem when solved with the chosen criteria considering the different levels of noise. The results shown are the average of the 5 optimum solutions for the 5 runs carried out.

Results demonstrate that the method is quite robust even with low damages and important intensities of noise, which makes the damage detection more accurate.

Figures 4 and 6 show the probability of damage existence (PDE) for the different elements and for levels of noise 5% and 15% using the procedure already commented.

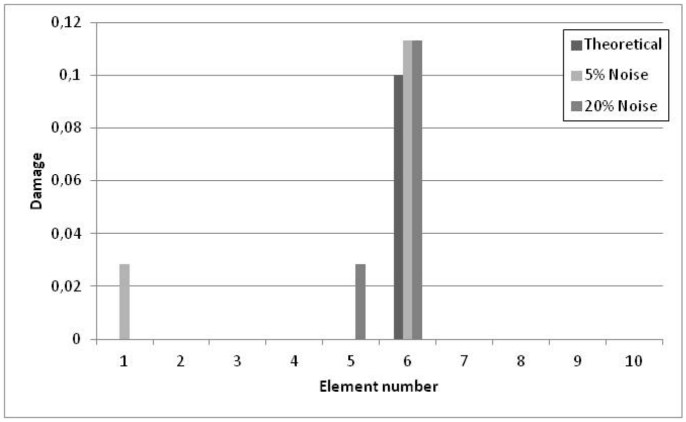

Figure 3. Damage distribution (simple scenario).

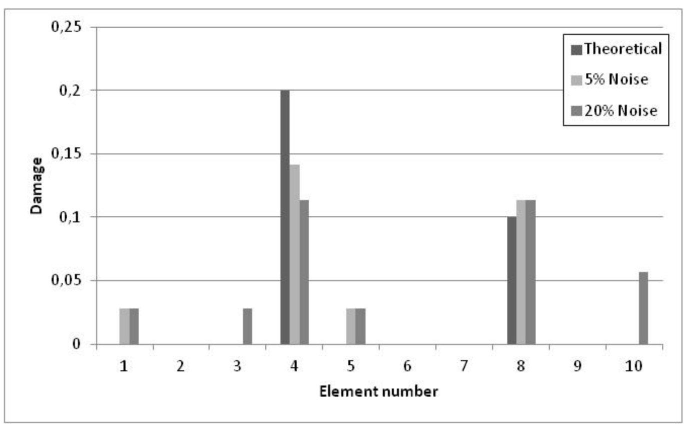

Figure 4. Damage distribution (multiple scenario).

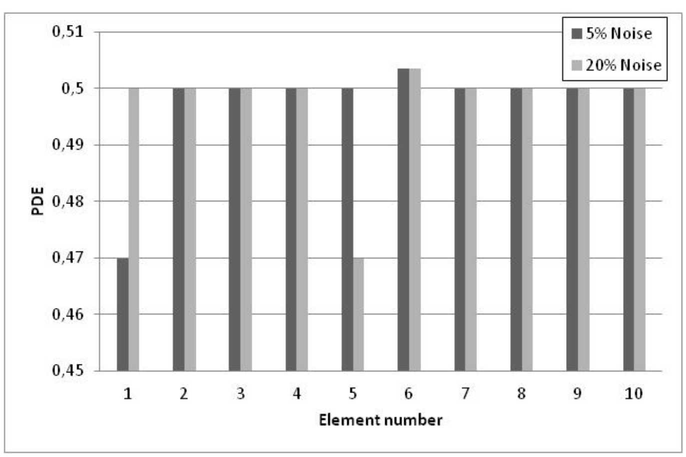

Figure 5. Probability of damage existence (simple scenario).

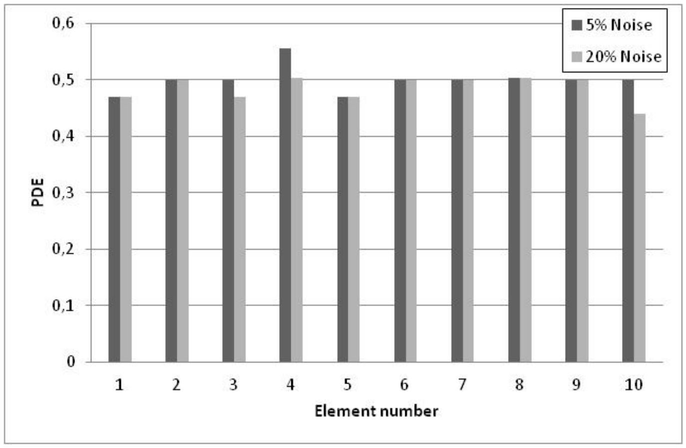

Figure 6. Probability of damage existence (multiple scenario).

CONCLUSIONS

A statistical multiobjective structural damage detection algorithm is developed in this paper. The approach has been focused in the performance and robustness of the damage identification procedure. The uncertainties existing in the structural model and measured structural modal parameters have been taken into account. The probability of damage existence has been obtained. The presented method has been verified by a numerical study on a simple supported beam with different levels of noise.

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REFERENCES

1. Y. Xia, H. Hao, Statistical damage identification of structures with frequency changes, Journal of Sound and Vibration **263** (2003) 853-870.
2. S. W. Doebling, C. R. Farrar, M. B. Prime, A summary review of vibration-based damage identification methods, *The Shock and Vibration Digest*. 30 (2) (1998) 91-105.
3. A. Konak, D. W. Coit, A. E. Smith, Multi-objective optimization using genetic algorithms: A tutorial, Reliability Engineering and System Safety **91** (2006) 992-1007.
4. R. Perera, A. Ruiz, C. Manzano, An evolutionary multiobjective framework for structural damage localization and quantification, Engineering Structures **29** (2007) 2540-2550.
5. Y. Jin, J. Branke, Evolutionary Optimization in Uncertain Environments – A Survey, IEEE Transactions on Evolutionary Computatios, Vol. 9, Nº 3, June 2005.
6. R. Perera, A. Ruiz, C. Manzano, Performance assessment of multicriteria damage identification genetic algorithms, Computers and Structures **87** (2009) 120-127.
7. R.J. Allemang, D.L. Brown, A correlation for modal vector analysis, *Proceedings of 1st international modal analysis conference* **110** (1982).
8. Babbar, M., Lakshmikantha, A., Goldberg, D.E. A modified NSGA-II to solve noisy multiobjective problems. In: Proceedings of Genetic and Evolutionary Computation Conference, Lecture Notes in Computer Handling Uncertainty in Indicator-Based Multiobjective Optimization, vol 2723. *Springer verlag*, Chicago, Illinois, USA, pp. 21 – 27 (2003).